2.2.42 From Torricelli's law, water in an open tank will flow out through a hole in the bottom at a speed equal to that it would acquire in a free-fall from the level of the water to the hole. A parabolic bowl has the shape of $y = x^2$, $0 \le x \le 1$, (units are feet) revolved around the y-axis. This bowl is initially full of water and at t = 0, a hole of radius ***a*** is punched at the bottom. How long will it take for the bowl to drain? HINT: An object dropped from height h will hit the ground at a speed of $v = \sqrt{(2gh)}$, where g is the gravitational constant. This formula is derived from equating the kinetic energy on impact $(1/2)mv^2$, with the work required to raise the object mgh.

Starting volume of rotated parabola:

$$y = x^2, \ |x| < 1$$

$$\int_0^1 \pi x^2 dy = \int_0^1 \pi y dy = \pi \frac{y^2}{2}\bigg|_{y=1} = \frac{\pi}{2}$$

Volume decreases as water runs out the hole of radius a at the bottom:

$$\frac{dV}{dt} = -\pi a^2 v = -\pi a^2 \sqrt{2gh}$$

Volume at any time:

$$V = \frac{\pi h^2}{2}, h = \sqrt{\frac{2V}{\pi}}, \frac{dV}{dt} = -\pi a^2 \sqrt{2gh} = -\pi a^2 \sqrt{2g\sqrt{\frac{2V}{\pi}}}$$

$$\int_{\frac{\pi}{2}}^{0} V^{\frac{-1}{4}} dV = \int_0^t -a^2 (2\pi)^{3/4} \sqrt{g} dt$$

$$0 - \frac{4}{3} V^{3/4}\bigg|_{V=\frac{\pi}{2}} = -a^2 (2\pi)^{3/4} \sqrt{g} \cdot t$$

$$t = \frac{\sqrt{2}}{3a^2\sqrt{g}}$$