2.3.17 A 2-foot length of 10-foot chain hangs off the end of a high table. Neglecting friction, find the time required for the chain to slide off the table. HINT: Model this problem with a second-order ODE and solve it using the following reduction of order technique: if ***x*** is the length of the chain hanging off the table and ***v = dx/dt*** then ***dv = dv/dx•dx/dt = v dv/dx***.

Assume the chain has uniform density = λ = m/10.
Then a length x hanging off the table has a mass = λx = mx/10.

$$m\frac{dv}{dt} = mv\frac{dv}{dx} = \frac{m}{10}xg$$

$$10\int v\,dv = g\int x\,dx = 32\int x\,dx$$

$$5v^2 = 16x^2 + C$$

$$at\ t = 0,\ v = 0,\ x = 2 \rightarrow C = -64$$

$$\frac{dx}{dt} = v = \sqrt{3.2x^2 - 12.8}$$

$$\int_2^{10} \frac{1}{\sqrt{3.2x^2 - 12.8}}dx = \frac{1}{\sqrt{3.2}}\int_2^{10} \frac{1}{\sqrt{x^2 - 4}}dx = \int_0^t dt$$

$$t = \frac{1}{\sqrt{3.2}}ln\left(\frac{x + \sqrt{x^2 - 4}}{2}\right)_2^{10} = \frac{1}{\sqrt{3.2}}ln\left(\frac{10 + \sqrt{96}}{2}\right) = 1.28\ s$$