

Math 275 Exam on Chapters 1-3

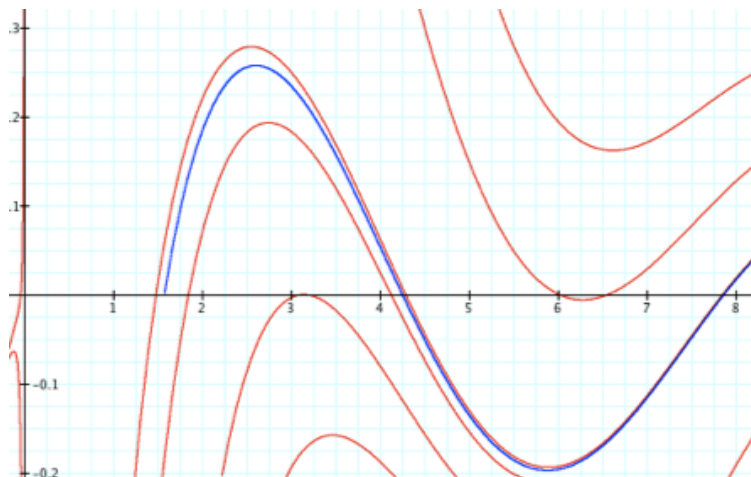
1. Solve the following initial-value problem and graph its solution.

$$x \frac{dy}{dx} + 2y = \sin(x), y\left(\frac{\pi}{2}\right) = 0$$

NOTE: It may be useful to know that

$$\int (x \sin x) dx = \sin x - x \cos x + C$$

$$y = \frac{\sin x}{x^2} - \frac{\cos x}{x} - \frac{1}{x^2}, x \geq \frac{\pi}{2}$$



2. Solve the following initial-value problem, graph its solution and determine its interval of existence.

$$(2xy^2 + 4x^3) dx + (2x^2y + 3) dy = 0, y(0) = 0$$

$$x^2y^2 + x^4 + 3y = 0$$

$$0 \leq x < \left(\frac{3}{2}\right)^{1/3}$$



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3. Consider the logistic model with harvesting applied to a fishing operation,

$$\frac{dy}{dt} = ay - cy^2 - h, y(0) = y_0$$

where a represents the growth rate, c the inhibitive factor, and h the harvesting (fishing) rate. Suppose that over the first year, the rate of change is such that $a = 0.6$, $c = 0.1$, $h = 0.5$, and $y_0 = 0.5$. Note that $y = 1$ corresponds to a thousand fish.

a) Draw the phase line.



b) Solve the equation to determine the size of the fish population at the end of 1 year and graph your solution over the interval $t = 0$ to $t = 1$ year.

$$y = \frac{5 - 9e^{-0.4x}}{1 - 9e^{-0.4x}}, x > 0$$



0.205

After the first year, fishing is prohibited, but parameters a and c remain the same.

c) Draw the phase line.

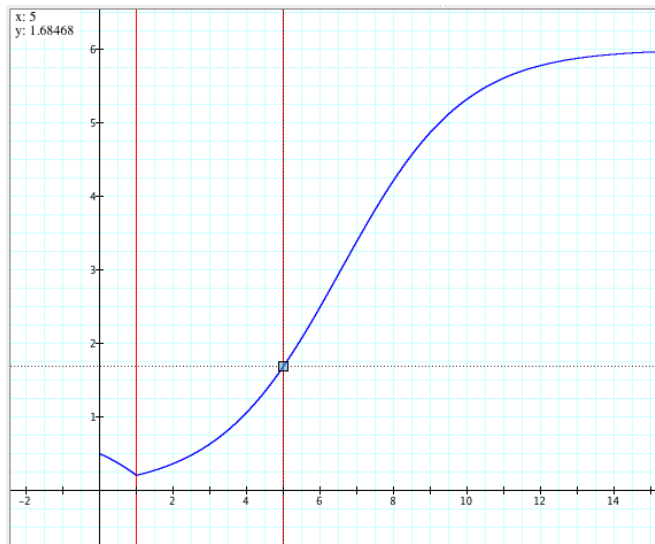
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d) Now solve the equation to determine the size of the fish population after 5 years and graph your solution over the interval $t = 1$ to $t = 15$ years.

$$y = \frac{6}{1 + 51.4493e^{-0.6x}}, x > 1$$

1.68468



4. Andre inherits \$80,000 from his grandfather's estate. The money is in an account that pays 6% annual interest, compounded continuously. The terms of the inheritance require that \$12000 be withdrawn each year for Andre's educational expenses. If no additional deposits are made, how long will the inheritance last before the funds are completely gone?

8.51 years