

***Practice Problem***

Given the initial-value problem:

$$y'' + 2ay' + by = 0, y(0) = 1, y'(0) = 0$$

- a) Rewrite the second-order equation as a system of two first-order equations.
- b) Write the system of two first-order equations as a matrix equation.
- c) What is the trace of the matrix in terms of  $a$  and  $b$ ?
- d) What is the determinant of the matrix in terms of  $a$  and  $b$ ?
- e) What are the eigenvalues of the matrix in terms of  $a$  and  $b$ ?
- f) Pick values  $a = 5$  and  $b = 16$ , identify the type of solution (saddle point, nodal sink, nodal source, spiral sink, spiral source or center), and make 3 graphs:  $y$  vs.  $t$ ,  $y'$  vs.  $t$ , and a phase plot ( $y'$  vs.  $y$ ).
- g) Pick values  $a = -5$  and  $b = 16$ , identify the type of solution, and make 3 graphs on a computer:  $y$  vs.  $t$ ,  $y'$  vs.  $t$ , and a phase plot ( $y'$  vs.  $y$ ).
- h) Pick values  $a = -1$  and  $b = 2$ , identify the type of solution, and make 3 graphs on a computer:  $y$  vs.  $t$ ,  $y'$  vs.  $t$ , and a phase plot ( $y'$  vs.  $y$ ).
- i) Pick values  $a = 1$  and  $b = 2$ , identify the type of solution, and make 3 graphs on a computer:  $y$  vs.  $t$ ,  $y'$  vs.  $t$ , and a phase plot ( $y'$  vs.  $y$ ).
- j) Pick values  $a = 3$  and  $b = -16$ , and identify the type of solution, and make 3 graphs on a computer:  $y$  vs.  $t$ ,  $y'$  vs.  $t$ , and a phase plot ( $y'$  vs.  $y$ ).
- k) Pick values  $a = 0$  and  $b = 1$ , and identify the type of solution, and make 3 graphs on a computer:  $y$  vs.  $t$ ,  $y'$  vs.  $t$ , and a phase plot ( $y'$  vs.  $y$ ).