## §8.2 - Integration by Parts

Integration by parts is a very powerful integration technique. It's based on the product rule for differentiation.

$$\frac{d}{dx}[uv] = u\,dv + v\,du$$

Integrating both sides of the equation above:

$$uv = \int u\,dv + \int v\,du$$

Solving for the first integral on the right:

$$\int u\,dv = uv - \int v\,du$$

*Example 1*

Evaluate the following integral:

$$\int xe^{-x}\,dx$$

Let

$$u = x \; and \; dv = e^{-x}dx$$

Then

$$du = dx \; and \; v = -e^{-x}$$

So,

$$\int u \, dv = uv - \int v \, du$$

$$\int xe^{-x} \, dx = -xe^{-x} - \int -e^{-x} \, dx = -xe^{-x} - e^{-x} + C$$

We can check our result by taking its derivative:

$$\frac{d}{dx}[-xe^{-x} - e^{-x} + C] = -e^{-x} + xe^{-x} + e^{-x} = xe^{-x}$$

*Example 2*

Evaluate the following integral:

$$\int ln \; x \, dx$$

Let u = ln x and dv = dx. Then du = (1/x) dx and v = x.

$$\int u \, dv = uv - \int v \, du$$

$$\int ln \; x \, dx = x \; ln \; x - \int x\left(\frac{1}{x}\right) dx = x \; ln \; x - x + C$$

*Example 3*

Evaluate the following integral:

$$\int x \ln x \, dx$$

*Example 4*

Evaluate the following integral:

$$\int x^2 \ln x \, dx$$

*Example 5*

Evaluate the following integral:

$$\int \arcsin x \, dx$$

## Integrals that Cycle

*Example 6*

Evaluate the following integral:

$$\int x^2 \sin x \, dx$$

*Example 7*

Evaluate the following integral:

$$\int e^{-x} \sin x \, dx$$